AP Calculus AB

Unit 9 - Applications of Differentiation

Worksheets



AP Calculus AB - Worksheet 73

In exercises 1-6, find the derivative.

1.
$$y = \int_{0}^{s} (\sin^{2} t) dt$$

2. $F(x) = \int_{2}^{s} (3t + \cos t^{2}) dt$
3. $g(x) = \int_{x}^{6} \ln(1+t^{2}) dt$
4. $F(x) = \int_{x}^{7} \sqrt{2t^{4} + t + 1} dt$
5. $y = \int_{x^{1}}^{3} \frac{\cos t}{t^{2} + 2} dt$
6. $y = \int_{\sin x}^{\cos x} t^{2} dt$
7. The graph of the function f shown consists of two line segments. Let g be the function given by $g(x) = \int_{0}^{5} f(t) dt$. Find
a) $g(-1)$
b) $g'(-1)$
c) $g''(-1)$
8. The continuous function f is defined on the interval $-4 \le x \le 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above. Let $g(x) = 2x + \int_{0}^{5} f(t) dt$. Find
a) $g(-3)$ and b) $g'(-3)$





* - optional problem

For #1-5

- a) Find and classify the critical point(s).
- b) Find the interval(s) where f(x) is increasing.
- c) Find the interval(s) where f(x) is decreasing.

1.
$$f(x) = 3x^2 - 3x + 2$$
 2. $f(x) = x^3 - x^2 - x$

 *3. $f(x) = 2x^3 - 9x^2 + 2$
 *4. $f(x) = \frac{x^4}{4} - x^3 + x^2$

 5. $f(x) = \frac{x-2}{x+2}$

6. Find *a* and *b* so that $f(x) = x^3 + ax^2 + b$ will have a critical point at (2,3).

7. Give the total number of maximum and minimum points of the function whose derivative is given by $f'(x) = x(x-3)^2(x+1)^4$.

For #8 and #9, use the graph of the function *f* to estimate where

- (a) f' = 0
- **(b)** f' < 0
- (c) f' > 0

Justify your answer.





Answers:

	Relative Min	Relative Max	Increasing	Decreasing
	f' changes from $-$ to $+$	f' changes from + to -	- f' > 0	f' < 0
1.	$\left(\frac{1}{2},\frac{5}{4}\right)$	None	$\left(\frac{1}{2},\infty\right)$	$\left(-\infty,\frac{1}{2}\right)$
2.	(1,-1)	$\left(-\frac{1}{3},\frac{5}{27}\right)$	$\left(-\infty,-\frac{1}{3}\right)\&\left(1,\infty\right)$	$\left(-\frac{1}{3},1\right)$
3	(3,-25)	(0,2)	$(-\infty,0)$ & $(3,\infty)$	(0,3)
4.	(0,0)&(2,0)	$\left(1,\frac{1}{4}\right)$	$(0,1)\&(2,\infty)$	$(-\infty,0)\&(1,2)$
5.	None	None	All real #s except $x = 2$	Never
6. $a = -3, b = 7$			one (min at $x = 0$)	
8. (a) $f'=0 @ x=-1 and x=1$			9. (a) $f'=0$ @ $x=-1$, $x=0$ and $x=1$	
(b) $f' < 0$ on $(-1,1)$			(b) $f' < 0$ on $(-\infty, -1)$ and $(0, 1)$	
(c) $f' > 0$ on $(-\infty, -1)$ and $(1, \infty)$			(c) $f' > 0$ on $(-1,0)$ and $(1,\infty)$	







- 3. The graph of the function f shown at left consists of two line segments. Let g be the function given by $g(x) = \int_0^x f(t) dt$.
 - a) Find the equation of the tangent line to g(x) at x = 0.

b) Determine the *x*-coordinate of each critical value on the graph of g(x). Classify each critical value as a local maximum, local minimum or neither. Justify your answers.

c) Determine the interval(s) on which g(x) is increasing. Justify your answer.

AP Calculus AB – Worksheet 77

In Exercises 1-5, find the average rate of change of the function over each interval.

1.
$$f(x) = x^3 + 1$$
 on [2,3]
2. $f(x) = \sqrt{4x+1}$ on [0,2]

3. $f(x) = e^x$ on [-2,0] **4.** $f(x) = \ln x$ on [100,103]

5.
$$f(x) = \sin x$$
 on $\left[\frac{\pi}{4}, \frac{3\pi}{4}\right]$

In Exercises 6-14, determine whether the Mean Value Theorem can be applied to f on the closed interval [a,b]. If it can be applied, find the value of c that satisfies $f'(c) = \frac{f(b) - f(a)}{b-a}$. If the Mean Value Theorem cannot be applied, explain why not.

6. $f(x) = x^2 - 2x - 2$ on $[-1,3]$	7. $f(x) = x^3 - x$ on [0,1]	8. $f(x) = \frac{x^2 - x - 6}{x - 1}$ on $[-2, 3]$
9. $f(x) = \sin x$ on $[0, \pi]$	10. $f(x) = \tan x$ on $(0, \pi)$	11. $f(x) = x^2 - 2x + 1$ on [1,3]
12. $f(x) = x^{\frac{3}{4}}$ on [0,16]	13. $f(x) = \sqrt{1 - x^2}$ on $[-1, 1]$	14. $f(x) = \frac{1}{x^2}$ on $[-1,1]$

2007 AP[®] CALCULUS AB FREE-RESPONSE QUESTIONS

x	f x	$f'\langle x \rangle$	g x	g' (x)
1	6	4	2	5
2	9	2	3	1
3	10	-4	4	2
4	-1	3	6	7

- 3. The functions f and g are differentiable for all real numbers, and g is strictly increasing. The table above gives values of the functions and their first derivatives at selected values of x. The function h is given by h(x) = f(g(x)) 6.
 - (a) Explain why there must be a value r for 1 < r < 3 such that h(r) = -5.
 - (b) Explain why there must be a value c for 1 < c < 3 such that h'(c) = -5.

**** Optional Problems**

For #1-3

- d) Find and classify the critical point(s).
- e) Find the interval(s) where f(x) is increasing.
- f) Find the interval(s) where f(x) is decreasing.

$1. f(x) = x^2 - x - 1$	2. $f(x) = 2x^4 - 4x^2 + 1$	**3. $f(x) = xe^x$
--------------------------	-----------------------------	--------------------

For # 4-6

- a) Find the *x*-coordinate of the point(s) of inflection.
- b) Find the interval(s) where f(x) is concave up.
- c) Find the interval(s) where f(x) is concave down.

4.
$$f(x) = 4x^3 + 21x^2 + 36x - 20$$

**5. $f(x) = 2x^{\frac{1}{5}} + 3$
6. $f(x) = -x^4 + 4x^3 - 4x + 1$

For #7-9, find all points of inflection of the function. Justify your answer.

7. $y = xe^x$	**8. $f(x) = \arctan x$	**9. $f(x) = x^{\frac{4}{3}} - 4x^{\frac{1}{3}}$
---------------	-------------------------	--



Let *f* be a function defined on the closed interval $-5 \le x \le 5$ with f(1) = 3. The graph of *f*', the derivative of *f*, consists of two semicircles and two line segments, as shown above.

- (a) For -5 < x < 5, find all values of x at which f has a relative maximum. Justify your answer.
- (b) For -5 < x < 5, find all values of x at which f has a point of inflection. Justify your answer.
- (c) Find all intervals on which the graph of f (not shown) is concave up. Justify your answer.
- (d) Find all intervals on which the graph of f (not shown) has a positive slope. Justify your answer.

Answers:

	Relative Min f' changes from $-$ to $+$	Relative Max f' changes from + to -	Increasing $f' > 0$	Decreasing $f' < 0$
1.	$\left(\frac{1}{2},-\frac{5}{4}\right)$	None	$\left(\frac{1}{2},\infty\right)$	$\left(-\infty,\frac{1}{2}\right)$
2.	(-1,-1) and $(1,-1)$	(0,1)	$(-1,0)$ and $(1,\infty)$	$(-\infty, -1)$ and $(0, 1)$
3	$\left(-1, -\frac{1}{e}\right)$	None	$(-1,\infty)$	$(-\infty, -1)$

	<i>x</i> -coordinate of point of inflection <i>f</i> " changes signs	Concave Up $f'' > 0$	Concave Down f'' < 0
4.	$x = -\frac{7}{4}$	$\left(-\frac{7}{4},\infty\right)$	$\left(-\infty,-\frac{7}{4}\right)$
5.	x = 0	$(-\infty,0)$	$(0,\infty)$
6.	x = 0 and $x = 2$	(0,2)	$(-\infty,2)$ and $(2,\infty)$

7. $\left(-2, -\frac{2}{e^2}\right)$ 8. $(0,0)$ 9. $(0,0)$ and $\left(-2, 6\sqrt[3]{2}\right)$	
---	--

Free Response Question:

- a) f has a relative maximum at x = -3 and x = 4 because f'(x) changes signs from positive to negative.
- b) f has a point of inflection at x = -4, -1, and x = 2 because f " changes signs
- c) f is concave up on (-5, -4) and (-1, 2) because f' is increasing or f'' > 0
- d) f has a positive slope on (-5, -3) and (1, 4) because f' > 0.

AP Calculus AB – Worksheet 79

For Problems #1-3, find the critical points of f(x) and use the **Second Derivative Test** to determine whether each corresponds to a local minimum or maximum

1	$f(x) = x^3 - 12x^2 + 45x$
2	$f(x) = x^4 - 8x^2 + 1$
3	$f(x) = \sin^2 x + \cos x, [0,\pi]$

4. Sketch a possible graph of f', the derivative of f, given the following characteristics.

- f(x) has local minimum values at x=1 and x=9.
- f(x) has local a maximum value at x=6.
- f(x) has points of inflection at x=3 and x=8.

5. Sketch a possible graph of f given the following characteristics.

- f''(x) > 0 on $(-\infty, 1)$ and $(4, \infty)$. - f''(x) < 0 on (1, 4).

6. Explain why the graph of $f(x) = x^4$ does not have any points of inflection.

2015AB 5



Graph of f'

The figure above shows the graph of f', the derivative of a twice-differentiable function f, on the interval [-3, 4]. The graph of f' has horizontal tangents at x = -1, x = 1, and x = 3. The areas of the regions bounded by the *x*-axis and the graph of f' on the intervals [-2, 1] and [1, 4] are 9 and 12, respectively.

- (a) Find all x-coordinates at which f has a relative maximum. Give a reason for your answer.
- (b) On what open intervals contained in -3 < x < 4 is the graph of *f* both concave down and decreasing? Give a reason for your answer.
- (c) Find the x-coordinates of all points of inflection for the graph of f. Give a reason for your answer.
- (d) Given that f(1) = 3, write an expression for f(x) that involves an integral. Find f(4) and f(-2).

1. Find *a* and *b* so that $f(x) = x^3 + ax^2 + b$ will have a critical point at (2,3).

2. Give the total number of max and min points of the function whose derivative is given by $f'(x) = x(x-3)^2(x+1)^4$.

3. Which point on the graph of *y* is y' < 0 and y'' < 0?



4. If $y = x^2 - 4x + 3$, where x can only be numbers such that $0 \le x \le 5$, then what is the largest value that y will be?

5. Find the *x*-values of the relative maximums and minimums for $y = 2x^3 - 3x^2 - 12x$.

6. Find all inflection points on the graph of $y = x^4 - 4x^3$.

7.
$$f(x) = x^4 - 4x^2$$
 has:(A) $1 \max, 2 \min$ (B) $1 \min, 2 \max$ (C) $1 \min, 1 \max$ (D) $2 \max, 0 \min$ (E) $2 \min, 0 \max$

If the graph of f(x) is given below for #8 - #11, give the letter for the point where:



Answers: 1. a = -3, b = 72. one (min at x = 0) 3. C 4. 8 5. Rel. max. at x = -1 Rel. min. at x = 26. Inflection Pts. (0,0)&(2,-16) 7. A

AP Calculus AB – Worksheet 81

1.	The difference of two numbers is 50. Find the two numbers so that their product is as small as possible.		
2.	Find two numbers whose sum is 36 and whose product is as large as possible.		
3.	A farmer has 100 feet of fencing and wishes to enclose a rectangular plot of land. The land borders a river, so no fencing is required on that side. What should the dimensions of the rectangle be in order to enclose the largest possible area.		
4.	A painter has enough paint to cover 600 square feet of area. He wishes to construct and then paint a square- bottomed box (height is unknown). What is the largest volume of the box that he can construct?		
5.	Find the absolute maximums and minimums for $y = 2x^3 - 3x^2 - 12x$ on $[-2,3]$.		
6.	Find all inflection points on the graph $y = x^4 - 4x^3$.		
7.	What is the total number of extrema of the function whose derivative is $f'(x) = x^2 (x+1)^3 (x-4)^3$?		
8.	Find all relative maximums and minimums for $y = x^3 - 3x + 1$.		
9.	The graph of f is given at right. Let $g(x) = \int_{1}^{x} f(t) dt$. a) Find the values of $g(3), g'(3)$ and $g''(3)$. b) Determine the interval on which the graph of g (not shown) is increasing. Justify your answer. c) Find the absolute maximum value of g on the interval $[-2, 4]$. Show the work that leads to your conclusion. d) Determine the x-coordinate(s) of any points of inflection on the graph of g. Justify your answer.		
10	If $F(x) = \int_0^{x^3} (2t-1)^2 dt$, then find $F'(x)$.		
11	For $y = \frac{x^2 + x}{x^2 - 7x - 8}$, give all <i>x</i> -coordinates at which the graph of $f(x)$ has discontinuities. Identify the discontinuities as either removable or non-removable.		

Answers:

1.	x = 25; y = -25
2.	x = 18; y = 18
3.	w = 25 ft.; $l = 50$ ft.
4.	1000 cubic feet
5	absolute maximum $(-1,7)$ & absolute minimum $(2,-20)$
6.	inflection point: $(0,0), (2,-16)$.
7.	2
0	Relative minimum: $(1, -1)$
8.	relative maximum $(-1,3)$

Given f(x) is a differentiable function.

Fill in the blanks to complete the table below

f(x)	f'(x)	f''(x)
f(x) is increasing		
	f'(x) < 0	
	f'(x) changes signs from positive to negative when $x = a$.	
f(x) has a local minimum when $x = a$.		
		f''(x) < 0
f(x) is concave up		
		f''(x) changes signs

The graph of f'(x) is given below. Sketch a possible graph of f(x).



4. A graph of f'(x), the derivative of f(x), is given below. The domain for f(x) is all real numbers.



- a) On what interval(s) is f(x) increasing? Explain.
- b) Determine the x-value of any local maximum on the graph of f(x) will occur. Justify your reasoning.

5. Sketch a graph of the function whose *derivative* satisfies the properties given in the table below.

x	(-∞, -1)	-1	(-1, 1)	1	(1, 3)	3	(3, ∞)
f'(x)	positive	0	negative	0	positive	0	negative

6. The accompanying figure shows the graph of the derivative of a function *f*. The domain of *f* is the closed interval $\begin{bmatrix} -3,3 \end{bmatrix}$.



- a) Identify and classify the *x*-coordinate of each critical value. Justify your answers.
- b) Determine the interval(s) on which f is increasing. Justify.
- c) Determine the interval(s) on which f is concave down. Justify your answers.
- 7. A farmer has 180 feet to enclose a rectangular field which is divided into two parts by his fence as shown the in figure below. Find the maximum area the farmer can enclose using his 180 feet of fencing.



- 8. A husband and wife have enough wire to construct 160 ft. of fence. They wish to use it to form three sides of a rectangular garden, one side of which is along a building. Find the dimensions that will yield the largest area.
- 9. We want to construct a cylindrical can with a bottom but no top that will have a volume of 30 cubic centimeters. Determine the dimensions of the can that will minimize the amount of material needed to construct the can.

AP Calculus AB - Worksheet 83

1.	Find all inflection points for $f(x) = 2x^3 + 6x^2 - 6x + 7$.				
2.	Find the maximum value of $y = -x^2 + 4x + 25$ on $[-2,3]$. (Find the y at the absolute maximum point)				
3.	Find the absolute minimum value (the <i>y</i> number) of $f(x) = x^3 - 6x^2$ on [1,2].				
y y	4. Which point on the graph of $f(x)$ is $f'(x) < 0$ and $f''(x) > 0$? 5. Which point on the graph of $f(x)$ is $f'(x) = 0$ and $f''(x) > 0$? 6. Which point on the graph of $f(x)$ is $f'(x) > 0$ and $f''(x) > 0$?				
7.	On what intervals is $f(x) = 2x^3 + 3x^2$ increasing?				
8.	A husband and wife have enough wire to construct 160 ft. of fence. They wish to use it to form three sides of a rectangular garden against the side of their house. Find the dimensions that will yield the largest area.				
9.	If the first derivative of <i>f</i> is negative for $x = 9$, which of the following statements must be true?				
	I. $f(9)$ is negative II. f has a minimum at $x = 9$ III. f is decreasing at $x = 9$				
	(A) I only (B) II only (C) III only (D) I & II (E) I & III				
10.	Find $F'(x)$ if $F(x) = \int_{4}^{x^{2}} 3\sin(t^{2}) dt$				
11.	• Find the absolute minimum point(s) of $f(x) = 2x^3 - 6x^2 + 1$ on the interval $[-2,3]$.				
12.	Find and classify all critical points for $f(x) = 2x^4 - 4x^2$. Justify your answer.				
13.	3. Find the total number of relative maximums of the function whose derivative is given by $f'(x) = x(x-2)^3(x+6)^5$. Justify your answer.				
14.	The graph of <i>f</i> is shown below. At which point is $f'(x) > 0$ and $f''(x) > 0$? f''(x) > 0? f''(x) > 0 f''(x) > 0? f''(x) > 0? Find all values of <i>x</i> for which the graph of $f(x) = x^4 - 8x^3 + 14$ is increasing. Justify your answer.				

16.	If $F(x) = \int_{2}^{2x} \ln(\cos(t^3)) dt$, find $F'(x)$.
17	$\frac{d}{dw} \int_{3}^{w} \sin t dt =$ A) $\cos w - \cos 3$ B) $-\cos w + \cos 3$ C) $-\cos w$ D) $\cos w$ E) $\sin w$

Answers

1) (-1,17)	2) 29	3) -16	4) A	5) E
6) B	7) $x < -1$ and $x > 0$	8) $w = 40$ $l = 80$	9) C	10) $F'(x) = 6x \sin x^4$
11) (-2,-39)	12)	13) 2 min; 1 max	14) G	15) (6,∞)
16) $F'(x) = 2\ln(\cos(8x^3))$	17) E			

Worksheet 83b



Answers:

$1_{2} \frac{dy}{dy} = -\frac{3_{1}}{1} \frac{ft}{ft}$	2. D	3. C
1. a. $\frac{dt}{dt} = \frac{1}{8} \sec t$		
$dA = 21 \text{ ft}^2$		
$\frac{1}{dt} = \frac{1}{16} \frac{1}{\text{sec}}$		
4. E	5. B	6. A